



AUM

American University Of The Middle East

Chapter 8

Interval Estimation

Confidence Intervals and Testing Hypothesis

	Mean (σ – Known)	Mean (σ – Unknown)	Proportion
Confidence Interval	$\left(\bar{x} - Z \frac{\sigma}{\sqrt{n}}, \bar{x} + Z \frac{\sigma}{\sqrt{n}} \right)$	$\left(\bar{x} - t \frac{s}{\sqrt{n}}, \bar{x} + t \frac{s}{\sqrt{n}} \right)$	$\left(\hat{p} - Z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + Z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$
Margin of Error	$Z \frac{\sigma}{\sqrt{n}}$	$t \frac{s}{\sqrt{n}}$	$Z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
Sample Size	$n = \left((Z \sigma) / E \right)^2$		$n = \left((Z \sqrt{\hat{p}(1-\hat{p})}) / E \right)^2$
Test Statistics	$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$	$T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$	$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$

CASE 1: MEAN (σ – *Known*)

Example: A simple random sample of 50 items from a population with standard deviation 6 results in a sample mean of 32.

(a) Construct a 90% confidence interval for the population mean.

$$n = 50, \sigma = 6, \alpha = 10\% = 0.1$$

$$\text{Step 1: } \frac{\alpha}{2} = 0.05$$

$$\text{Step 2: } Z_{0.05} = \frac{1.64+1.65}{2} = 1.645$$

$$\text{C.I: } \left(\bar{X} - Z \frac{\sigma}{\sqrt{n}}, \bar{X} + Z \frac{\sigma}{\sqrt{n}} \right) = \left(32 - (1.645) \frac{6}{\sqrt{50}}, 32 + (1.645) \frac{6}{\sqrt{50}} \right) = (30.6, 33.4)$$

(b) Construct a 95% confidence interval for the population mean.

$$n = 50, \sigma = 6, \alpha = 5\% = 0.05$$

$$\text{Step 1: } \frac{\alpha}{2} = 0.025$$

$$\text{Step 2: } Z_{0.025} = 1.96$$

$$\text{C.I: } \left(\bar{X} - Z \frac{\sigma}{\sqrt{n}}, \bar{X} + Z \frac{\sigma}{\sqrt{n}} \right) = \left(32 - (1.96) \frac{6}{\sqrt{50}}, 32 + (1.96) \frac{6}{\sqrt{50}} \right) = (30.34, 33.66)$$

(c) Construct a 99% confidence interval for the population mean.

$$n = 50, \sigma = 6, \alpha = 1\% = 0.01$$

$$\text{Step 1: } \frac{\alpha}{2} = 0.005$$

$$\text{Step 2: } Z_{0.005} = \frac{2.57+2.58}{2} = 2.575$$

$$\text{C.I: } \left(\bar{X} - Z \frac{\sigma}{\sqrt{n}}, \bar{X} + Z \frac{\sigma}{\sqrt{n}} \right) = \left(32 - (2.575) \frac{6}{\sqrt{50}}, 32 + (2.575) \frac{6}{\sqrt{50}} \right) = (29.82, 34.19)$$

(d) What is the margin of error in each case a, b, c?

$$ME_{(a)} = (1.645) \frac{6}{\sqrt{50}} = 1.3958$$

$$ME_{(b)} = (1.96) \frac{6}{\sqrt{50}} = 1.6631$$

$$ME_{(c)} = (2.575) \frac{6}{\sqrt{50}} = 2.185$$

(e) How large a sample is needed if the desired margin of error is 0.5 for the confidence interval of 90%?

$$n = \left(\frac{Z\sigma}{E} \right)^2 = \left(\frac{(1.645)(6)}{0.5} \right)^2 = 389.6676 \sim 390$$

CASE 2: MEAN (σ – *Unknown*)

Example: A simple random sample of 24 provided as a sample mean of 22.5 and a sample standard deviation of 4.4;

(a) Construct a 90% confidence interval for the population mean.

$$n = 24, \bar{X} = 22.5, s = 4.4, \alpha = 10\% = 0.1$$

$$\text{Step 1: } \frac{\alpha}{2} = 0.05$$

$$\text{Step 2: } t_{0.05, 23} = 1.714$$

$$\text{C.I: } \left(\bar{X} - t \frac{s}{\sqrt{n}}, \bar{X} + t \frac{s}{\sqrt{n}} \right) = \left(22.5 - (1.714) \frac{4.4}{\sqrt{24}}, 22.5 + (1.714) \frac{4.4}{\sqrt{24}} \right) = (20.96, 24.039)$$

(b) Construct a 95% confidence interval for the population mean.

$$n = 24, \bar{X} = 22.5, s = 4.4, \alpha = 5\% = 0.05$$

$$\text{Step 1: } \frac{\alpha}{2} = 0.025$$

$$\text{Step 2: } t_{0.025, 23} = 2.069$$

$$\text{C.I: } \left(\bar{X} - t \frac{s}{\sqrt{n}}, \bar{X} + t \frac{s}{\sqrt{n}} \right) = \left(22.5 - (2.069) \frac{4.4}{\sqrt{24}}, 22.5 + (2.069) \frac{4.4}{\sqrt{24}} \right) = (20.64, 24.35)$$

(c) Construct a 99% confidence interval for the population mean.

$$n = 24, \bar{X} = 22.5, s = 4.4, \alpha = 1\% = 0.01$$

$$\text{Step 1: } \frac{\alpha}{2} = 0.005$$

$$\text{Step 2: } t_{0.005, 23} = 2.807$$

$$\text{C.I.: } \left(\bar{X} - t \frac{s}{\sqrt{n}}, \bar{X} + t \frac{s}{\sqrt{n}} \right) = \left(22.5 - (2.807) \frac{4.4}{\sqrt{24}}, 22.5 + (2.807) \frac{4.4}{\sqrt{24}} \right) = (19.97, 25.02)$$

(d) What is the margin of error in each case a, b, c?

$$ME_{(a)} = (1.714) \frac{4.4}{\sqrt{24}} = 1.5394$$

$$ME_{(b)} = (2.069) \frac{4.4}{\sqrt{24}} = 1.8583$$

$$ME_{(c)} = (2.807) \frac{4.4}{\sqrt{24}} = 2.5211$$

CASE 3: PROPORTION

Example: In a sample of 162 companies, 104 reported profits that beat the prior estimates, 29 reported profits that matched the prior estimates, 29 reported profits that fell short of the prior estimates.

(a) Produce a point estimate for the proportion of companies that fell short of estimates.

$$\frac{29}{162} = 0.179 = \hat{p}$$

(b) Calculate the margin of error for a confidence interval of %95.

$$n = 162, \alpha = 5\% = 0.05$$

$$\text{Step 1: } \frac{\alpha}{2} = 0.025$$

$$\text{Step 2: } Z_{0.025} = 1.96$$

$$ME = Z \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = (1.96) \sqrt{\frac{0.179(1 - 0.179)}{162}} = 0.059$$

(c) Construct a %95 confidence interval for the companies which fell short of the prior estimates.

$$\text{C.I.: } (\hat{p} - ME, \hat{p} + ME) = (0.179 - 0.059, 0.179 + 0.059) = (0.12, 0.238)$$